

The importance of flavor in leptogenesis

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ABSTRACT: We study leptogenesis from the out-of-equilibrium decays of the lightest heavy neutrino N_1 in the medium (low) temperature regime, $T \lesssim 10^{12}$ GeV (10^9 GeV), where the rates of processes mediated by the τ (and μ) Yukawa coupling are non negligible, implying that the effects of lepton flavors must be taken into account. We find important quantitative and qualitative differences with respect to the case where flavor effects are ignored: (i) The cosmic baryon asymmetry can be enhanced by up to one order of magnitude; (ii) The sign of the asymmetry can be opposite to what one would predict from the sign of the total lepton asymmetry ϵ_1 ; (iii) Successful leptogenesis is possible even with $\epsilon_1 = 0$.

KEYWORDS: Baryogenesis, Cosmology of Theories beyond the SM.

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1. Introduction

A very attractive mechanism for explaining the origin of the baryon asymmetry of the Universe ($Y_B \equiv (n_B - \bar{n}_B)/s \simeq 8.7 \times 10^{-11}$) is baryogenesis through leptogenesis [1, 2]. Leptogenesis scenarios naturally appear within the standard model minimally extended to include the see-saw mechanism [3], because all the conditions [4] required for generating a cosmic lepton asymmetry are generically satisfied in the decays of the see-saw related heavy singlet neutrinos: (i) The Majorana nature of their masses is a source of lepton number violation; (ii) Complex Yukawa couplings induce CP violation in the interference between the tree level and loop decay amplitudes; (iii) For a heavy Majorana mass scale $\sim 10^{11\pm 3}$ GeV, sizable deviations from thermal equilibrium in the primeval expanding Universe can occur at the time the heavy neutrinos decay. Partial conversion of the lepton asymmetry into a baryon asymmetry then proceeds by means of anomalous $B + L$ -violating electroweak sphaleron interactions [5] that are standard model processes. Qualitatively it is then almost unavoidable that a lepton (and hence a $B - L$) asymmetry is induced in the decays of the see-saw singlet neutrinos and, since no standard model reaction violates $B - L$, this asymmetry survives until the present epoch. The question of whether leptogenesis is able to explain the puzzle of the baryon asymmetry of the Universe is then a quantitative one.

The quantitative analysis of leptogenesis has become more and more sophisticated in recent years, taking into account many subtle but significant ingredients, such as various

washout effects [6–10], thermal corrections to particle masses and CP violating asymmetries [11], and spectator processes [12, 13]. The latter are $B - L$ conserving processes, such as standard model gauge interactions, some Yukawa interactions involving the heaviest fermions, and electroweak and strong non-perturbative ‘sphaleron’ interactions. These processes do not participate directly in the generation or washout of the asymmetries (hence the name ‘spectator’), but have important effects in determining the asymmetric densities of the various particles, mainly by imposing certain relations among the chemical potentials of different particle species [12]. A detailed analysis of how spectator processes affect the washout back-reactions and concur to determine the final value of the baryon asymmetry has been recently presented in [13].

In spite of all these refinements, one potentially very significant aspect of leptogenesis has been only rarely addressed [14–16], and that is flavor. Neglecting flavor issues is, however, justified only if the process of leptogenesis is completed at a rather high temperature, $T > 10^{12}$ GeV. A reliable computation of the lepton and baryon asymmetries when leptogenesis occurs in the intermediate or low temperature windows must include flavor effects. In this paper, while taking into account all the effects discussed in [13], we introduce in the analysis additional important phenomena that have to do with the flavor composition of the lepton states involved in leptogenesis.¹ In particular, we focus on the decoherence effects that are induced by the charged lepton Yukawa interactions on the lepton doublets produced in the decays of the heavy neutrinos. As soon as these Yukawa interactions approach equilibrium, they act essentially as measuring devices that project all the lepton densities onto the flavor basis. Lepton number asymmetries and washout effects then become flavor dependent, and this can lead to a final baryon asymmetry that is different in size and possibly even sign from the one that would arise if flavor issues were irrelevant.

The plan of this paper is as follows. In section 2 we present the main physics ideas that underlie the most important effects of flavor in leptogenesis, and derive the relevant results in a qualitative way. In section 3 we present the network of flavor dependent Boltzmann equations, including all the spectator processes discussed in [13]. In section 4 we separately analyze each of the relevant temperature regimes: we impose the appropriate equilibrium conditions and discuss their implications for the network of Boltzmann equations. We also present results for a set of representative flavor structures and for different temperature regimes. In section 5 we compare our results to what is obtained when flavor effects are neglected or irrelevant, and we explain the main mechanism underlying the large enhancements of $B - L$ that flavor effects can induce. In section 6 we summarize our main results.

2. The basic ideas

In this section we study leptogenesis within temperature ranges well below 10^{13} GeV, where lepton flavor issues can have an important impact on the way leptogenesis is realized. A

¹In our previous work [13] we imposed certain flavor alignment conditions, whereby the effects discussed in this paper become irrelevant.

proper treatment of the decay and scattering processes occurring in a thermal bath that consists of a statistical mixture of various flavor states should be carried out within a density matrix formalism, as discussed for example in [14]. Here we follow a simpler approach that is based on a physically intuitive formulation of the problem, and allows us to obtain all the qualitative features of the possible solutions. The results we derive here agree well with what will be obtained in section 4 by solving numerically the detailed flavor-dependent Boltzmann equations that are derived in section 3.

2.1 Flavor CP violating effects

The heavy singlet neutrinos N_α needed for the see-saw model decay into lepton and antilepton doublets, implying lepton number violation. We denote the lepton doublets produced in the N_α decays with the same index $\alpha = 1, 2, 3$:²

$$\begin{aligned}\Gamma_\alpha &\equiv \Gamma(N_\alpha \rightarrow \ell_\alpha H), \\ \bar{\Gamma}_\alpha &\equiv \Gamma(N_\alpha \rightarrow \bar{\ell}'_\alpha \bar{H}).\end{aligned}$$

CP -violation in N_α decays can manifest itself in two different ways:

1. Leptons and antileptons are produced at different rates,

$$\Gamma_\alpha \neq \bar{\Gamma}_\alpha. \tag{2.1}$$

2. The leptons and antileptons produced in N_α decays are not CP conjugate states,

$$CP(\bar{\ell}'_\alpha) \equiv \ell'_\alpha \neq \ell_\alpha. \tag{2.2}$$

If the rate of charged lepton Yukawa interactions is much slower than the rate of the heavy neutrino ($N-\ell$) Yukawa interactions, flavor issues can be neglected since, regardless of their flavor composition, ℓ_α and $\bar{\ell}'_\alpha$ remain coherent states between two successive interactions. In this case only the effect in eq. (2.1) is important. Indeed, leptogenesis studies have generally concentrated on this first effect. The situation is, however, different if leptogenesis occurs when the processes mediated by the τ (and possibly μ) charged lepton Yukawa couplings are faster than the $N-\ell$ Yukawa interactions (a sufficient condition for this is that these processes occur at a rate comparable to the expansion rate of the Universe). Then, ℓ_α and $\bar{\ell}'_\alpha$ are no longer the interacting states populating the thermal bath and ℓ_i and $\bar{\ell}_i$ with $i = \tau, (\mu, e)$ should be considered instead. The second effect in (2.2) can then become of major importance for the generation of cosmic asymmetries.

The amount of lepton asymmetry produced per N_α decay ($\alpha = 1, 2, 3$) is

$$\epsilon_\alpha = \frac{\Gamma_\alpha - \bar{\Gamma}_\alpha}{\Gamma_\alpha + \bar{\Gamma}_\alpha}. \tag{2.3}$$

In order that non-vanishing ϵ_α asymmetry would arise, the two sets of three states ℓ_α and ℓ'_α cannot form an orthogonal basis because, to get CP violation from loops, it is required that the lepton doublet coupled to the external N_α (the decaying heavy neutrino) couples

²The subindex in $\ell_{1,2,3}$ bears no relation to the neutrino mass eigenstates $\nu_{1,2,3}$.

also to a virtual N_β (with $\beta \neq \alpha$) appearing in the loop. Assuming a hierarchical pattern for the heavy N_α masses M_α , and given the non-orthogonality of the $\ell_{1,2,3}^{(\prime)}$ states, the most natural situation is then that any asymmetry produced in $N_{2,3}$ decays is quickly erased by fast L violating interactions involving N_1 . One can still envisage a situation in which $\ell_3 \not\perp \ell_2$ is responsible for $\epsilon_2 \neq 0$, but an approximate orthogonality $\ell_2 \perp \ell_1$ prevents ϵ_2 from being washed out by the N_1 lepton number violating processes. In this case a lepton asymmetry produced in N_2 decays can survive until all the L violating processes involving N_1 freeze-out. However, if this freeze-out occurs after lepton flavor dynamics has become important, then all the $\ell_\alpha^{(\prime)}$ will be effectively projected onto $\ell_{\tau,(\mu,e)}$ flavor states. Then additional ‘flavor alignment’ conditions are necessary in order to preserve the ϵ_2 asymmetry, as for example $\ell_1 \propto \ell_e$ and $\ell_{2,3} \perp \ell_e$. Models that realize this scenario have been studied for example in [14, 17, 18]. In the following we disregard this possibility, and concentrate on the decay of the lightest heavy neutrino N_1 (and on ϵ_1) as the dominant source of a cosmic lepton asymmetry.

If during leptogenesis the (required) deviations from thermal equilibrium are not large, inverse decays and other washout processes become important to determine the amount of ϵ_1 asymmetry that can be effectively converted into a baryon asymmetry by the $B + L$ violating electroweak sphaleron processes. In the cases where the N_1 heavy neutrinos have an initial thermal abundance, or when thermal abundance is reached due to inverse decays and other scattering processes, it is customary to express the present density of cosmic baryon asymmetry relative to the entropy density s as follows:

$$\frac{n_B}{s} = -\kappa_s \epsilon_1 \eta. \tag{2.4}$$

The numerical factor $\kappa_s \simeq 1.38 \times 10^{-3}$ accounts for the $B - L$ entropy dilution from the leptogenesis temperature down to the electroweak breaking scale, as well as for the electroweak sphalerons $B - L \rightarrow B$ conversion factors. The factor η , that can range between 0 and 1, is the so called efficiency (or washout) factor and accounts for the fraction of lepton asymmetry surviving the washout processes. If N_1 decays occur strongly out of equilibrium, then all back-reactions are negligible and $\eta \approx 1$. On the other hand, values of $\eta \sim 10^{-2} - 10^{-3}$ that are typical of strong washout regimes, when deviations from thermal equilibrium are mild, can still yield successful leptogenesis while ensuring at the same time that n_B/s is largely independent of initial conditions.

When $M_1 \gg 10^{12}$ GeV, successful leptogenesis requires that the $N_1 - \ell_1$ couplings are sizable and, in particular, larger than all the charged lepton Yukawa couplings. Then in the relevant temperature window, around $T \approx M_1$, charged lepton Yukawa processes are much slower than the processes involving N_1 (and also slower than the rate of the Universe expansion). In this regime, the composition of the two states ℓ_1 and $\bar{\ell}'_1$ in terms of the lepton flavor states ℓ_i ($i = \tau, \mu, e$) is irrelevant since the doublet states produced in the decays keep coherence between two different scatterings involving N_1 . However, if leptogenesis occurs at lower temperatures, processes mediated by the τ Yukawa coupling (and for $T \lesssim 10^9$ GeV also the processes mediated by the μ Yukawa coupling) become faster than the reactions involving N_1 . Then ℓ_1 and $\bar{\ell}'_1$ lose their coherence between two

subsequent L -violating interactions. Before they can rescatter in reactions involving N_1 , they are projected onto the lepton flavor basis with respective probabilities for each flavor i :

$$K_{1i} = |\langle \ell_1 | \ell_i \rangle|^2 \quad \text{and} \quad \bar{K}_{1i} = |\langle \bar{\ell}'_1 | \bar{\ell}_i \rangle|^2 \quad (CP(\bar{\ell}_i) = \ell_i). \quad (2.5)$$

In the following, we drop the index 1 in K_{1i} , \bar{K}_{1i} (as well as in the decay rates $\Gamma_1, \bar{\Gamma}_1$), leaving understood that we always refer to N_1 related quantities. We will also concentrate exclusively on the medium and low temperature regimes, in which the charged lepton Yukawa interactions effectively ‘measure’, at least in part, the flavor composition of $\ell_1^{(\prime)}$. We distinguish the following possibilities:

1. Alignment: if the $i = \tau$ (or $i = \tau, \mu$) Yukawa processes are in equilibrium, but both ℓ_1 and ℓ'_1 are aligned or orthogonal to one flavor i ($K_i = \bar{K}_i = 1$ or 0), leptogenesis reduces to a one-flavor problem, much like the unflavored case of the high temperature regimes. Different flavor alignments mainly affect just the way in which the lepton asymmetry gets distributed between lepton left- and right-handed states (the latter being sterile with respect to all L -violating processes) and this yields numerical effects not larger than a few tens of percent. A detailed analysis of the various aligned cases has been recently presented in [13].
2. Non-alignment, with only the τ (but not the μ and e) Yukawa processes in equilibrium: the μ and e flavor components of ℓ_1 and $\bar{\ell}'_1$ are not disentangled from each other during leptogenesis. It is then convenient to introduce two combinations of the μ and e flavors, ℓ_b and $\bar{\ell}'_b$, that are orthogonal to, respectively, ℓ_1 and $\bar{\ell}'_1$, thus implying $K_b = \bar{K}_b = 0$. The two combinations of μ and e , ℓ_a and $\bar{\ell}'_a$, that are orthogonal to, respectively, ℓ_b and $\bar{\ell}'_b$, satisfy $K_a = 1 - K_\tau$ and $\bar{K}_a = 1 - \bar{K}_\tau$ (in general $CP(\bar{\ell}'_{a,b}) \neq \ell_{a,b}$). In this case, even in the absence of any particular alignment condition, leptogenesis can be treated as an effectively two-flavor problem.
3. Non-alignment, and both τ and μ Yukawa interactions fast: all the lepton flavor components of $\ell_1^{(\prime)}$ are effectively resolved, and the thermal bath is populated by the (CP conjugate) flavor states ℓ_i and $\bar{\ell}_i$ ($i = e, \mu, \tau$). The full three-flavor problem has to be considered in this case. (The onset of thermal equilibrium for the electron Yukawa coupling occurs at temperatures too low to be relevant for standard leptogenesis scenarios, but would not add qualitative changes to this picture).

In the rest of this paper we address the lepton flavor issues for leptogenesis in the above cases 2 and 3. We show how these issues have a significant impact on the way leptogenesis is realized in the intermediate and low temperature regimes.

2.2 Lepton flavor asymmetries

The CP asymmetry for N_1 decays into the ℓ_j lepton flavor is defined as

$$\epsilon_1^j = \frac{\Gamma(N_1 \rightarrow \ell_j H) - \bar{\Gamma}(N_1 \rightarrow \bar{\ell}_j \bar{H})}{\Gamma + \bar{\Gamma}} \equiv \frac{\Gamma_j - \bar{\Gamma}_j}{\Gamma + \bar{\Gamma}}. \quad (2.6)$$

Since, by definition, $K_j = \Gamma_j/\Gamma$ and $\bar{K}_j = \bar{\Gamma}_j/\bar{\Gamma}$, we can conveniently express ϵ_1^j as

$$\epsilon_1^j = \frac{\Gamma K_j - \bar{\Gamma} \bar{K}_j}{\Gamma + \bar{\Gamma}} = \frac{K_j + \bar{K}_j}{2} \epsilon_1 + \frac{K_j - \bar{K}_j}{2} \simeq \epsilon_1 K_j^0 + \frac{\Delta K_j}{2}. \quad (2.7)$$

Here ϵ_1 is the total asymmetry in N_1 decays defined in eq. (2.3), $\Delta K_j \equiv K_j - \bar{K}_j$, and $K_j^0 \equiv \Gamma_j^0/\Gamma^0$ represents the ratio of the (CP conserving) tree level decay amplitudes, with $K_j^0 = \bar{K}_j^0$. In the last equality in eq. (2.7), the first term, proportional to ϵ_1 , corresponds to CP violating effects of the first type [eq. (2.1)], the second term, proportional to ΔK_j , is an effect of the second type [eq. (2.2)] since it vanishes when $CP(\ell'_1) = \ell_1$, and higher order terms of $\mathcal{O}(\Delta K_j \cdot \epsilon_1)$ are neglected.

For temperature regimes where the flavor states ℓ_j are the ones relevant for leptogenesis, rather than the ℓ_α states, eq. (2.4) should be replaced with

$$\frac{n_B}{s} = -\kappa_s \sum_{j=1}^{n_f} \epsilon_1^j \eta_j, \quad (2.8)$$

where $n_f = 2(3)$ is the relevant number of ‘active’ flavors in the medium (low) temperature regime. In these regimes, off-shell $\Delta L = 2$ washout processes involving in particular $N_{2,3}$ are generally negligible, and hence all the relevant washout processes involve just the heavy neutrino N_1 and are therefore associated with the flavor projectors K_j . Since, as will be discussed in more detail in section 5, the efficiency factors are, to a good approximation, inversely proportional to the washout rates, we can write $\eta_j \simeq \min(\eta/K_j^0, 1)$, where η represents the washout factor one would obtain neglecting flavor effects. The value of η_j saturates to unity when $K_j^0 \approx \eta$ and, in the cases we have studied numerically, this typically occurs for $K_j^0 \approx \text{few} \times 10^{-2}$. In these cases, one can think of the decay of $N_1 \rightarrow \ell_j H$ as one that proceeds much like strongly out-of-equilibrium decays. However, if $K_j^0 \ll 1$, the condition $\sum_i K_i^0 = 1$ implies that for (one or two of the) other flavors, the back reactions are rather fast and in particular can be quite effective in populating the N_1 states, so as to keep their abundance close to thermal during a relevant part of the leptogenesis era. As in the standard thermal leptogenesis scenarios, this ensures independence from initial conditions and, moreover, that a sizable amount of the asymmetry, much larger than a fraction K_j^0 , can end up surviving in the ℓ_j flavor. Inserting eq. (2.7) into eq. (2.8) gives:

$$\frac{n_B}{s} \approx -\kappa_s \begin{cases} n_f \epsilon_1 \eta + \eta \sum_i \frac{\Delta K_i}{2K_i} & K_i^0 \gtrsim \eta \quad \text{for all } i \\ \eta(n_f - 1)\epsilon_1 + K_j^0 \epsilon_1 + \eta \sum_{i \neq j} \frac{\Delta K_i}{2K_i} + \frac{\Delta K_j}{2} & K_j^0 \lesssim \eta, \quad K_{i \neq j}^0 > \eta \end{cases} \quad (2.9)$$

We note the following:

1. In the second line of this equation, that corresponds to the situation in which the value of η_j saturates to ≈ 1 , the first three terms are suppressed at least as η , while the last term is not.
2. When $K_j^0 \ll \eta$, an alignment condition is approached [13]. Flavor effects become strongly suppressed here when all ΔK_i vanish, as is always the case in the two flavor situations (see section 4).

3. If $\Delta K_i = 0$ for all i , that is in the absence of CP violating effects of the type of eq. (2.2), and if $K_i^0 > \eta$ for all i , that is away from alignment conditions, then the final baryon asymmetry is always enhanced by a factor n_f with respect to the case in which flavor effects are irrelevant (or are neglected). This enhancement occurs independently of the particular values of the K_i^0 , as is clearly seen from figure 1, and it holds also with respect to the aligned cases discussed in [13] since alignment effectively enforces the condition $n_f = 1$. It is easy to understand the reason for the n_f enhancement. When the total decay rates are projected onto the flavor i , each flavor asymmetry gets reduced compared to the total asymmetry: $\epsilon_1^i \sim \epsilon_1 K_i^0$. However, this is compensated by a suppression of the corresponding washout factor, $\eta_i \sim \eta/K_i^0$. Then the sum over flavors yields the n_f enhancement.
4. When ℓ_1 has approximately equal projections onto the different ℓ_i doublets, implying that $K_i^0 \approx 1/n_f$ for all i , the result of the previous point still holds, regardless of the particular values $\Delta K_i \neq 0$. This can be easily seen from the first line of eq. (2.9) by noting that $\sum_i \Delta K_i = 0$.
5. In the general situation, the terms proportional to ΔK_i do not vanish. Let us stress that $|\Delta K_i/\epsilon_1|$, being the ratio of two (higher order) CP violating quantities, is not constrained to particularly small values and can well be sizeably bigger than unity. This means that the effects of $CP(\bar{\ell}) \neq \ell$ can be the dominant ones in determining the size of the final baryon asymmetry. Moreover, since the signs of the ΔK_i are not directly related to the sign of ϵ_1 , it is clear that one cannot infer in a model independent way the sign of the cosmic baryon asymmetry only on the basis of ϵ_1 . This situation (dominance of the ΔK_j effect) is even more likely when one K_j^0 is very small and $\eta_j \approx 1$. Then, as can be seen from the second line in eq. (2.9), the term $\Delta K_j/2$ being not suppressed by the small value of η can easily dominate over all other terms.

2.3 Dependence on lagrangian parameters

We now proceed to express the various relevant quantities — ϵ_1 , ϵ_1^j , ΔK_j and K_j^0 — in terms of the lagrangian parameters. In the mass eigenbasis of the heavy neutrinos N_α and of the charged leptons (e_i denote the SU(2) singlet charged leptons), the leptonic Yukawa and mass terms read:

$$\mathcal{L}_Y = -\frac{1}{2}M_\alpha \bar{N}_\alpha^c N_\alpha^c - (\lambda_{\alpha i} \bar{N}_\alpha \ell_i \tilde{H}^\dagger + h_i \bar{e}_i \ell_i H^\dagger + \text{h.c.}). \quad (2.10)$$

Explicit computation of the vertex and self-energy contributions to ϵ_1^j with this lagrangian yields [19]:

$$\epsilon_1^j = \frac{-1}{8\pi(\lambda\lambda^\dagger)_{11}} \sum_{\beta \neq 1} \text{Im} \left\{ \lambda_{\beta j} \lambda_{1j}^* \left[\frac{3}{2\sqrt{x_\beta}} (\lambda\lambda^\dagger)_{\beta 1} + \frac{1}{x_\beta} (\lambda\lambda^\dagger)_{1\beta} \right] \right\}, \quad (2.11)$$

where $x_\beta = M_\beta^2/M_1^2$. The coefficient of $(\lambda\lambda^\dagger)_{\beta 1}$ within square brackets in eq. (2.11) comes from the leading term in the expansion for $x_\beta \gg 1$ of

$$\frac{\sqrt{x_\beta}}{1-x_\beta} + \sqrt{x_\beta} \left(1 - (1+x_\beta) \ln \frac{1+x_\beta}{x_\beta} \right) = -\frac{3}{2}x_\beta^{-1/2} - \frac{5}{6}x_\beta^{-3/2} + \mathcal{O}(x_\beta^{-5/2}). \quad (2.12)$$

The first term in the l.h.s. originates from self-energy type of loop corrections, while the second term corresponds to the proper vertex correction. The coefficient of the second term $(\lambda\lambda^\dagger)_{1\beta}$ in square brackets in (2.11) comes from the self-energy type of diagram with ‘inverted’ direction of the fermion line in the loop, and corresponds to the leading term in the expansion $(1-x_\beta)^{-1} = -x_\beta^{-1} + \mathcal{O}(x_\beta^{-2})$. Eq. (2.11) holds when the mass splittings between the heavy neutrino masses are much larger than the decay widths, $M_\beta - M_1 \gg \Gamma_{\beta,1}$. The lowest order expression for the flavor projectors reads:

$$K_j^0 = \bar{K}_j^0 = \frac{\lambda_{1j}\lambda_{1j}^*}{(\lambda\lambda^\dagger)_{11}}. \quad (2.13)$$

By summing eq. (2.11) over the flavor index we obtain the total asymmetry:

$$\epsilon_1 = \frac{-3}{16\pi(\lambda\lambda^\dagger)_{11}} \sum_{\beta \neq 1} \text{Im} \left\{ \frac{1}{\sqrt{x_\beta}} (\lambda\lambda^\dagger)_{\beta 1}^2 \right\}. \quad (2.14)$$

We do not need to separately calculate ΔK_j in terms of the lagrangian parameters since, from eq. (2.7), $\Delta K_j/2 = \epsilon_1^j - \epsilon_1 K_j^0$. As was first noted in ref. [19], the term $(\lambda\lambda^\dagger)_{1\beta}$ contributes to the flavor asymmetry ϵ_1^j in eq. (2.11), but — since $(\lambda\lambda^\dagger)_{\beta 1}(\lambda\lambda^\dagger)_{1\beta}$ is real — it does not contribute to the total asymmetry ϵ_1 in eq. (2.14). We conclude therefore that it corresponds to a CP violating effect of the second type.

In the basis that we are using, which is defined by diagonal matrices for the heavy neutrino masses M_α and for the charged lepton Yukawa couplings h_j , the $\lambda_{\alpha j}$ couplings can be conveniently written in the Casas-Ibarra parametrization [20]:

$$\lambda_{\alpha j} = \frac{1}{v} \left[\sqrt{M} R \sqrt{m} U^\dagger \right]_{\alpha j} \quad (2.15)$$

where $v = \langle H \rangle$ is the Higgs vacuum expectation value, $M = \text{diag}(M_1, M_2, M_3)$ is the diagonal matrix of the heavy masses, $m = \text{diag}(m_1, m_2, m_3)$ is the diagonal matrix of the light neutrino masses, $R = v M^{-1/2} \lambda U m^{-1/2}$ is an orthogonal complex matrix ($R^T \cdot R = I$) and U is the leptonic mixing matrix. With this parametrization, the term $(\lambda\lambda^\dagger)_{\beta 1}^2$ that controls the total asymmetry in eq. (2.14) is given by

$$(\lambda\lambda^\dagger)_{\beta 1}^2 = \frac{M_1 M_\beta}{v^4} \left(\sum_i m_i R_{1i}^* R_{\beta i} \right)^2, \quad (2.16)$$

while the term $(\lambda_{\beta j} \lambda_{1j}^*) (\lambda\lambda^\dagger)_{\beta 1}$, that gives the leading contribution to ϵ_1^j in eq. (2.11), is given by

$$(\lambda_{\beta j} \lambda_{1j}^*) (\lambda\lambda^\dagger)_{\beta 1} = \frac{M_1 M_\beta}{v^4} \left(\sum_i m_i R_{1i}^* R_{\beta i} \right) \left(\sum_{k,l} \sqrt{m_k m_l} R_{\beta l} R_{1k}^* U_{jl}^* U_{jk} \right). \quad (2.17)$$

Eqs. (2.16) and (2.17) show that, if in the basis where the light and heavy neutrino Majorana masses are real, R were a *real* orthogonal matrix, two very intriguing consequences would follow:

1. The total asymmetry would vanish, $\epsilon_1 = 0$, while, in general, $\epsilon_1^j \neq 0$. This is a surprising possibility for leptogenesis.
2. The asymmetries ϵ_1^j , which involve the imaginary part of eq. (2.17), would depend only on the CP violating phases present in U , which (in principle) are measurable in low energy experiments.

This situation would be quite different from the one usually considered in leptogenesis scenarios, in which the final baryon asymmetry, being proportional to ϵ_1 , depends through the matrix R on all the low energy and high energy parameters. It remains to be seen if a real matrix R can naturally arise in some model.

3. The network of Boltzmann equations

We consider the scenario in which the heavy neutrino masses are hierarchical, $M_1 \ll M_{2,3}$, and consequently the lepton asymmetry is generated mainly via the CP and lepton number violating decays of the lightest singlet neutrino N_1 to the lepton doublets ℓ_1 and $\bar{\ell}'_1$. The important processes involving these states are the following (all the γ 's below denote the thermally averaged rates):

- N_1 decays and inverse decays, with rates $\gamma_D = \gamma(N_1 \leftrightarrow \ell_1 H)$ and $\bar{\gamma}_D = \gamma(N_1 \leftrightarrow \bar{\ell}'_1 \bar{H})$.
- $\Delta L = 1$ Higgs-mediated scattering processes with rates such as $\gamma_{S_s} = \gamma(\ell_1 N_1 \leftrightarrow Q_3 \bar{t})$ and $\gamma_{S_t} = \gamma(\ell_1 Q_3 \leftrightarrow N_1 t)$, where Q_3 and t are respectively the third generation quark doublet and the top $SU(2)$ singlet, as well as those involving gauge bosons, such as in $\ell_1 N_1 \rightarrow HA$ (with $A = W^{3,\pm}$ or B). CP violating effects might be numerically important in these $2 \leftrightarrow 2$ scatterings [8, 9]. However, they do not add qualitatively new features to the analysis. Therefore we neglect CP violation in all these processes and accordingly we use the tree level expression for the flavor projectors \bar{K}_j , $K_j \simeq K_j^0$.
- The s -channel scattering processes $\gamma_{N_s} = \gamma(\ell_1 H \leftrightarrow \bar{\ell}'_1 \bar{H})$ with on-shell N_1 are already accounted for by decays and inverse decays. When the lepton doublets are projected onto the flavor basis, subtraction of these rates to avoid double counting must be carried out with care.
- The off-shell scatterings $\gamma_{N_s}^{sub}$, involving the (pole subtracted) s -channel and the u -channel, as well as the t -channel scatterings $\gamma_{N_t} = \gamma(\ell \ell \leftrightarrow \bar{H} \bar{H})$, depend on all the K_j^α projectors in a rather complicated way. However, since they are subdominant in the temperature ranges we are interested in, they can be safely neglected.

In the temperature regimes in which the charged lepton Yukawa couplings become non-negligible ($T \ll 10^{13}$ GeV), the corresponding interactions define a flavor basis for the

lepton doublets. Then the decay rates and scattering processes involving the specific flavors ℓ_i and anti-flavors $\bar{\ell}_i$ have to be considered, and the Boltzmann equations should track the evolution of all the relevant single-flavor asymmetries. To obtain the appropriate set of equations one can rely on the density operator approach discussed in [14]. One defines a density matrix ρ as the difference between the density matrices for the leptons and for the antileptons, so that $\rho_{ii} \propto Y_{L_i}$, and normalizes ρ so that $\sum_i \rho_{ii} = Y_L$. A very useful property of ρ is that the off-diagonal terms, ρ_{ij} with $i \neq j$ (the coherences) vanish whenever the Yukawa interactions of one of the flavor states ℓ_i or ℓ_j are in thermodynamic equilibrium, since this effectively projects the leptonic states present in the thermal bath onto those aligned or orthogonal to the flavors characterized by non-negligible Yukawa interactions.³ Another property that we use is that if the population of one state vanishes, $\rho_{ii} = 0$, then all the coherences ρ_{ij} associated with this state also vanish (this follows from e.g. the inequality $\rho_{ii}\rho_{jj} \geq |\rho_{ij}|^2$). These properties allow one to restrict the general equation for ρ to a subset of equations for the relevant flavor diagonal directions $\rho_{ii} = Y_{L_i}$.

Following the approach outlined in [13], in writing down the Boltzmann equations we account for all the particle densities that are relevant to the washout processes. Moreover, in the evolution equations for the lepton flavor asymmetries we also include the term $dY_{L_i}^{EW}/dz$ that formally accounts for the fact that electroweak sphalerons constitute an additional source of lepton flavor violation. Then, for consistency, we also need to add the equation $dY_B/dz = dY_B^{EW}/dz$ to account for baryon number violation by the sphaleron processes. Given that sphaleron interactions preserve the three charges $\Delta_i \equiv B/3 - L_i$ associated to anomaly-free currents, it follows that $Y_B^{EW}/3 = Y_{L_i}^{EW}$. By subtracting the equations for the lepton flavor densities from the equation for baryon number weighted by a suitable factor $1/3$, we obtain the following network of flavor dependent Boltzmann equations:

$$\frac{dY_{N_1}}{dz} = \frac{-1}{sHz} \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) (\gamma_D + 2\gamma_{Ss} + 4\gamma_{St}), \tag{3.1}$$

$$\begin{aligned} \frac{dY_{\Delta_i}}{dz} = \frac{-1}{sHz} \left\{ \left[\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \epsilon_1^i - \frac{1}{2}(y_{\ell_i} + y_H) K_i^0 \right] \gamma_D \right. \\ \left. - \left[2y_{\ell_i} + (y_t - y_{Q_3}) \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} + 1 \right) \right] K_i^0 \gamma_{St} - \left[\frac{Y_{N_1}}{Y_{N_1}^{eq}} y_{\ell_i} + y_t - y_{Q_3} \right] K_i^0 \gamma_{Ss} \right\}, \tag{3.2} \end{aligned}$$

where we have used the standard notation $z \equiv M_1/T$. In these equations, $Y_{N_1} \equiv n_{N_1}/s$ denotes the density of the lightest heavy neutrino (with two degrees of freedom) relative to the entropy s , $y_X \equiv (n_X - n_{\bar{X}})/n_X^{eq}$ denote the asymmetries for the various relevant species $X = \ell, H, t, Q_3$ and all the asymmetries are normalized to the Maxwell-Boltzmann

³The transition region between the regimes where a specific lepton Yukawa coupling is completely negligible, and the one in which it mediates reactions in full thermal equilibrium, that is when Yukawa reactions for one specific flavor are approaching equilibrium, should be treated with care, since off diagonal entries ρ_{ij} in the density matrix might not be dumped fast enough to be neglected in the flavor dynamics [21]. However, within the temperature ranges in which the lepton Yukawa reactions for each flavor are fully in equilibrium or strongly out of equilibrium, our Boltzmann equations can be safely applied.

equilibrium densities. Notice that Y_{Δ_i} is the Δ_i number density, also normalized to the entropy, with $Y_{L_i} = (2y_{\ell_i} + y_{e_i})Y^{eq}$. The reaction rates are summed over initial and final state quantum numbers, including the gauge multiplicities. In the asymmetries y_X , $X = \ell, H$ or Q_3 label any of the two doublet components, not their sum, and hence we normalize y_X to the equilibrium densities with just one degree of freedom. This is different from other conventions (e.g. the one used in [11]) and allows us to keep the proportionality $y_X \propto \mu_X$ in terms of the chemical potentials, with the usual convention that e.g. μ_{ℓ_i} is the chemical potential of each one of the two SU(2) components of the doublet ℓ_i . As already said, the subdominant $\Delta L = 2$ off-shell scatterings, and CP violation in ($\Delta L = 1$) $2 \leftrightarrow 2$ processes [8, 9] have been neglected in eq. (3.2), and we make two further simplifications:

1. In eqs. (3.1)–(3.2) and in what follows we ignore finite temperature corrections to the particle masses and couplings [11]. In particular we take all equilibrium number densities n_X^{eq} equal to those of massless particles.
2. We ignore scatterings involving gauge bosons, for whose rates no consensus has been achieved so far [11, 8]. They do not introduce qualitatively new effects and no further density asymmetries are associated to them.

We would like to emphasize the following points regarding eq. (3.2):

1. The washout terms are controlled by the K -projectors. The sources of the asymmetry, on the other hand, receive additional contributions from the ΔK 's. Note that ΔK_i is not simply proportional to the respective K_i (see eq. (2.7)). This has important consequences: when, for at least one flavor, $\Delta K_i/\epsilon_1 \gtrsim K_i^0$, the results are qualitatively different from the case where, for all the flavors, $\Delta K_i/\epsilon_1 \ll K_i^0$. In particular, the fact that the sign of ΔK_i can be opposite to that of the $K_i^0\epsilon_1$ term opens up the possibility that leptonic asymmetry-densities of opposite sign are generated.
2. Subtraction of the on-shell contributions from the s -channel N_1 heavy neutrino exchange, that corresponds to $\Delta L = 2$ s -channel scatterings, has to be performed with care. The cross sections are flavor dependent: $\gamma_{N_s}^{\Delta L_i=2}(\ell_i H \rightarrow \bar{\ell}_i \bar{H})$ changes L_i by two units, but other scatterings, $\gamma_{N_s}^{\Delta L_i=1}(\ell_i H \rightarrow \sum_{j \neq i} \bar{\ell}_j \bar{H})$ change L_i by only one unit. Furthermore, differently from the unflavored case, the $\Delta L = 0$ ($\Delta L_i = 1$) channels, $\gamma_{N_s}^{\Delta L_i=1}(\ell_i H \rightarrow \sum_{j \neq i} \ell_j H)$, together with their asymmetries, must also be taken into account. Moreover, through inverse processes the asymmetries in the flavors $j \neq i$ do affect the evolution of y_{ℓ_i} (this is similar to the way y_H, y_{Q_3} and y_t act in the proper $\Delta L_i = 1$ channels in the second line of eq. (3.2)). Nevertheless, we see from eq. (3.2) that the result of the subtraction agrees with what one would obtain by naively generalizing from the case of the flavor independent equation (see e.g. ref. [11]) to the flavor dependent case.
3. Consider the case $K_i = \bar{K}_i = 1$ (and hence $K_{j \neq i} = \bar{K}_{j \neq i} = 0$). Then we have $Y_{\Delta_{j \neq i}} = 0$. Since $Y_{B-L} = \sum_k Y_{\Delta_k}$, the equation for Y_{Δ_i} coincides (as expected) with that for Y_{B-L} in the unflavored case, or in the cases with flavor alignment, see eq. (13) in ref. [13].

4. As a consequence of the sum-rules $\sum_i K_i = \sum_i \bar{K}_i = 1$ (that imply $\sum_i \Delta K_i = 0$ and $\sum_i \epsilon_1^i = \epsilon_1$), the equation for $Y_{B-L} = \sum_k Y_{\Delta_k}$ has a simple form also in the general case $K_i \neq \bar{K}_i \neq 1$: Similarly to the unflavored or aligned cases [13], this equation depends on ϵ_1 as the source term, and on a single asymmetry-density $\tilde{y}_\ell \equiv \sum_i K_i^0 y_{\ell_i}$. The weighted sum of asymmetries, \tilde{y}_ℓ , represents the effective lepton doublet asymmetry coupled to the washout of Y_{B-L} . Of course, all the complications related with the flavor structure are now hidden in \tilde{y}_ℓ whose detailed evolution is determined by the additional equations that still depend explicitly on K_i^0 and ϵ_1^i . However, as we will see, there is always a point in the K -space for which $\tilde{y}_\ell \propto Y_{B-L}$. In this particular situation, the equation for Y_{B-L} decouples from the other equations and can provide a simple representative one-flavor approximation to the flavor dependent case, that still captures some of the main effects of flavor.

The network of equations eq. (3.1) and (3.2) can be solved after the densities y_{ℓ_i} (or \tilde{y}_ℓ), y_H and $y_t - y_{Q_3}$ are expressed in terms of the quantities Y_{Δ_j} with the help of the equilibrium conditions imposed by the fast reactions, as described in the next section. The value of $B - L$ at the end of the leptogenesis era obtained by solving the Boltzmann equations remains subsequently unaffected until the present epoch. If electroweak sphalerons go out of equilibrium before the electroweak phase transition, the present baryon asymmetry is given, assuming a single Higgs doublet, by the relation [22]

$$n_B = \frac{28}{79} n_{B-L}. \tag{3.3}$$

If, instead, electroweak sphalerons remain in equilibrium until slightly after the electroweak phase transition (as would be the case if, as presently believed, the electroweak phase transition was not strongly first order) the final relation between B and $B - L$ would be somewhat different [23].

4. The equilibrium conditions

In this section we discuss the equilibrium conditions that hold in the different temperature regimes which can be relevant to study flavor effects in leptogenesis. Since leptogenesis takes place during the out of equilibrium decay of the lightest heavy right-handed neutrino N_1 , i.e. at temperatures $T \sim M_1$, the relevant constraints that have to be imposed among the different particle densities depend essentially on the value of M_1 . To allow for a straightforward estimate of the importance of flavor, we chose the relevant temperature windows as in ref. [13], where flavor effects were irrelevant because of the imposition of alignment constraints, and also in presenting the results we follow closely that analysis. We use the equilibrium conditions specific of each temperature regime to express y_{ℓ_i} , y_H and $y_t - y_{Q_3}$ in terms of the Y_{Δ_j} 's.

4.1 General considerations

The number density asymmetries for the particles n_X entering in eq. (3.2) are related to

the corresponding chemical potentials through

$$n_X - n_{\bar{X}} = \frac{g_X T^3}{6} \begin{cases} \mu_X/T & \text{fermions,} \\ 2\mu_X/T & \text{bosons,} \end{cases} \quad (4.1)$$

where g_X is the number of degrees of freedom of X . For any given temperature regime the specific set of reactions that are in chemical equilibrium enforce algebraic relations between different chemical potentials [22]. In the entire range of temperatures relevant for leptogenesis, the interactions mediated by the top-quark Yukawa coupling h_t , and by the $SU(3) \times SU(2) \times U(1)$ gauge interactions, are always in equilibrium. Moreover, at the intermediate-low temperatures in which flavor effects can be important, strong QCD sphalerons are also in equilibrium. This situation has the following consequences:

- Equilibration of the chemical potentials for the different quark colors is guaranteed because the chemical potentials of the gluons vanish, $\mu_g = 0$.
- Equilibration of the chemical potentials for the two members of an $SU(2)$ doublet is guaranteed by the vanishing, above the electroweak phase transition, of $\mu_{W^+} = -\mu_{W^-} = 0$. This condition was implicitly implemented in eq. (3.2) where we used $\mu_Q \equiv \mu_{u_L} = \mu_{d_L}$, $\mu_\ell \equiv \mu_{e_L} = \mu_{\nu_L}$ and $\mu_H \equiv \mu_{H^+} = \mu_{H^0}$ to write the particle number asymmetries directly in terms of the number densities of the $SU(2)$ doublets.
- Hypercharge neutrality implies

$$\sum_i (\mu_{Q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i}) + 2\mu_H = 0, \quad (4.2)$$

where u_i , d_i and e_i denote the $SU(2)$ singlet fermions of the i -th generation.

- The equilibrium condition for the Yukawa interactions of the top-quark $\mu_t = \mu_{Q_3} + \mu_H$ yields:

$$y_t - y_{Q_3} = \frac{y_H}{2}, \quad (4.3)$$

where the factor $1/2$ arises from the relative factor of 2 between the number asymmetry and chemical potential for the bosons, see eq. (4.1).

- Because of their larger rates, QCD sphalerons equilibration occurs at higher temperatures than for the corresponding electroweak processes, presumably around $T_s \sim 10^{13}$ GeV [24–26]) and in any case long before equilibrium is reached for the τ Yukawa processes. This implies the additional constraint

$$\sum_i (2\mu_{Q_i} - \mu_{u_i} - \mu_{d_i}) = 0. \quad (4.4)$$

The condition in eq. (4.3) allows one to rewrite the r.h.s. of eq. (3.2) in terms of only the two asymmetries y_{ℓ_i} and y_H . To express these asymmetries in terms of the Y_{Δ_i} we define two matrices C^ℓ and C^H through the relations:

$$y_{\ell_i} = - \sum_j C_{ij}^\ell \frac{Y_{\Delta_j}}{Y_{eq}}, \quad y_H = - \sum_j C_j^H \frac{Y_{\Delta_j}}{Y_{eq}}. \quad (4.5)$$

The matrices C^ℓ and C^H constitute a generalization to the case of flavor non-alignment of the coefficients c_ℓ and c_H , introduced in [13]. The numerical values of their entries are determined by the constraints among the various chemical potentials enforced by the fast reactions that are in equilibrium in the temperature range ($T \sim M_1$) where the asymmetries are produced. The Boltzmann equations in eq. (3.2) can now be rewritten as follows:

$$\begin{aligned} \frac{dY_{\Delta_i}}{dz} = & \frac{-1}{sHz} \left\{ \left(\frac{Y_N}{Y_N^{eq}} - 1 \right) \epsilon_1^i \gamma_D \right. \\ & + K_i^0 \sum_j \left[\frac{1}{2} (C_{ij}^\ell + C_j^H) \gamma_D + \left(\frac{Y_N}{Y_N^{eq}} - 1 \right) \left(C_{ij}^\ell \gamma_{Ss} + \frac{C_j^H}{2} \gamma_{St} \right) \right. \\ & \left. \left. + \left(2 C_{ij}^\ell + C_j^H \right) \left(\gamma_{St} + \frac{1}{2} \gamma_{Ss} \right) \right] \frac{Y_{\Delta_j}}{Y^{eq}} \right\}. \end{aligned} \quad (4.6)$$

These equations are general enough to account for all the effects of the relevant spectator processes (Yukawa interactions, electroweak and QCD sphalerons) as well as for a general lepton flavor structure.

The fact that the consequences of flavor cannot be easily read off from the system of coupled equations (4.6) impedes a simple comparison with the results of the unflavored cases. We later show the results obtained by numerically integrating the set of coupled equations. But, in order to get some insight into the results, it is possible to introduce an approximation to the general equations (4.6) in the form of a one-flavor equation for Y_{B-L} that accounts quite accurately for the numerical impact of flavor effects for two classes of models:

1. Models in which N_1 decays with approximately equal rates to all flavors ($K_i^0 \approx 1/n_f$ for all ℓ_i).
2. Models in which all the flavor asymmetries ϵ_1^i are dominated by the term $K_i^0 \epsilon_1$ [see eq. (2.7)].

From the discussion in section 2.2 it is clear that both kind of models have the common feature of being sensitive only to CP violating effects of the type in eq. (2.1). The approximation to be discussed below captures in full this type of effects, but it is blind to the CP violating effects of the second type in eq. (2.2), and therefore does not yield reliable results for the cases where the asymmetries are dominated by the effects of $\Delta K \neq 0$.

We proceed as follows. We consider particular flavor structures leading to K_i^0 values that satisfy the conditions $\sum_i K_i^0 C_{ij}^\ell = \tilde{c}_\ell$, independently of the value of $j = 1, \dots, n_f$. We further introduce for the Higgs asymmetry an average coefficient $\tilde{c}_H \equiv \sum_j C_j^H / n_f$ and neglect the terms $\delta C_j^H = C_j^H - \tilde{c}_H$. (In all the cases that we consider $\delta C_j^H / \tilde{c}_H \lesssim 15\%$). Then, for this particular configuration, we can add the equations (4.6) over the flavors i to obtain an equation for Y_{B-L} that is independent of flavor indexes:

$$\begin{aligned} \frac{dY_{B-L}}{dz} = & \frac{-1}{sHz} \left\{ \left(\frac{Y_N}{Y_N^{eq}} - 1 \right) \epsilon_1 \gamma_D \right. \\ & \left. + \frac{Y_{B-L}}{Y^{eq}} \left[\frac{1}{2} (\tilde{c}_\ell + \tilde{c}_H) \gamma_D + \left(\frac{Y_N}{Y_N^{eq}} - 1 \right) \left(\tilde{c}_\ell \gamma_{Ss} + \frac{\tilde{c}_H}{2} \gamma_{St} \right) \right] \right\} \end{aligned}$$

$$+ (2\tilde{c}_\ell + \tilde{c}_H) \left(\gamma_{St} + \frac{1}{2}\gamma_{Ss} \right) \Big] \Big\}. \quad (4.7)$$

Of course, it is useful to proceed in this way only to the extent that this special case is representative of a more general class of flavor configurations (corresponding e.g. to different sets of K_i). As we will see, this is indeed true for the two classes of models mentioned above. Since eq. (4.7) is similar in form to the one studied in ref. [13], where different coefficients c_ℓ and c_H corresponding to situations of flavor alignment were introduced, the values of \tilde{c}_ℓ and \tilde{c}_H will give, by direct comparison, a measure of the possible impact of flavor effects in these models.

4.2 Specific temperature ranges and flavor structures

We now discuss the flavor effects in the various relevant temperature ranges. In order to clearly show the impact of these effects, we conduct our analysis in a way that allows for a meaningful comparison with our results in [13] in which - due to appropriate alignment conditions - flavor issues played no role. Thus we use the same temperature windows (below $T \sim 10^{13}$) as in [13], and (obviously) impose the same equilibrium conditions.

As explained above, some of our main results can be understood from the specific, effectively one-flavor, cases that are described by eq. (4.7). These cases are presented in table 1 and can be easily compared with the corresponding results in table 1 of [13]. In the last column in table 1 we give the resulting $B - L$ asymmetry. In the fourth and fifth columns, we give the values of \tilde{c}_ℓ and \tilde{c}_H . We remind the reader that the sum $\tilde{c}_\ell + \tilde{c}_H$ gives a crude scaling of the overall strength of the washout processes, while the ratio $\tilde{c}_H/\tilde{c}_\ell$ gives a rough estimate of the relative contribution of the Higgs asymmetry to the washout.

For more general flavor configurations that do not belong to the two classes of models 1 and 2 above, and for which the approximation in eq. (4.7) does not hold, we present numerical results in a graphic form in figures 1 and 2. To disentangle the impact of the various effects from that of the input parameters, the $B - L$ asymmetry is calculated in all cases with fixed values of $\tilde{m}_1 = 0.06$ eV and $M_1 = 10^{11}$ GeV, where $\tilde{m}_1 \equiv v^2(\lambda\lambda^\dagger)_{11}/M_1$ determines the departure from equilibrium of the heavy neutrino N_1 and provides an overall scale for the strength of the washout processes. The value $\tilde{m}_1 = 0.06$ eV is intermediate between the regime in which departures from equilibrium are large and all washout effects are generally negligible ($\tilde{m}_1 < 10^{-3}$ eV) and the regime in which washout processes are so efficient that often the surviving baryon asymmetry is too small ($\tilde{m}_1 \gtrsim 0.1$ eV). The choice of this value is also motivated by the atmospheric neutrino mass-squared difference if neutrino masses are hierarchical. As concerns M_1 , it is clear that the relevant temperature range is actually determined by it, yet we fix the value at $M_1 = 10^{11}$ GeV in order to have a meaningful comparison of the various effects of interest. Namely, since in each regime considered the same asymmetries are produced in the decay of the heavy neutrinos, a comparison between the final values of $B - L$ for the different cases can be directly interpreted in terms of suppressions or enhancements of the washout processes. We assume an initially vanishing value for Y_N and for all the particles density-asymmetries, but for $\tilde{m}_1 > 10^{-2}$ eV the results are insensitive to the initial values. The values of the parameters adopted here as well as the initial conditions are the same as in [13].

Equilibrium processes, constraints, coefficients and $B - L$ asymmetry

	T (GeV)	Equilibrium	Constraints	\tilde{c}_ℓ	\tilde{c}_H	$\frac{ Y_{B-L} }{10^{-5}\epsilon_1}$	
$B = 0$	$10^{12 \div 13}$	h_b, h_τ	$\left. \begin{aligned} b &= Q_3 - H, \\ \tau &= \ell_\tau - H \end{aligned} \right\}$	$K_\tau^0 = \frac{16}{27}$	$\frac{2}{9}$	$\frac{7}{32}$	1.2
	$10^{11 \div 12}$	+ EW-Sph	$\sum_i (3Q_i + \ell_i) = 0$	$K_\tau^0 = \frac{4}{7}$	$\frac{6}{35}$	$\frac{97}{460}$	1.4
$B \neq 0$	$10^{8 \div 11}$	+ h_c, h_s, h_μ	$\left. \begin{aligned} c &= Q_2 + H, \\ s &= Q_2 - H, \\ \mu &= \ell_\mu - H \end{aligned} \right\}$	$K_{\tau,\mu}^0 = \frac{19}{53}$	$\frac{5}{53}$	$\frac{47}{358}$	2.5
	$\ll 10^8$	all Yukawas h_i		$K_{\tau,\mu,e}^0 = \frac{1}{3}$	$\frac{7}{79}$	$\frac{8}{79}$	3.0

Table 1: The temperature window is given in the first column, the relevant interactions in equilibrium in the second, and the constraints on the chemical potential in the third. (Chemical potentials are labeled with the same notation used for the fields: $\mu_{Q_i} = Q_i$, $\mu_{\ell_i} = \ell_i$ for the $SU(2)$ doublets, $\mu_{u_i} = u_i$, $\mu_{d_i} = d_i$, $\mu_{e_i} = e_i$ for the singlets and $\mu_H = H$ for the Higgs.) We also give in the third column the K_i^0 -values that are compatible with eq. (4.7). The values of the coefficients \tilde{c}_ℓ and \tilde{c}_H are given in, respectively, the fourth and fifth column while the resulting $B - L$ asymmetry (in units of $10^{-5} \times \epsilon_1$) obtained for $\tilde{m}_1 = 0.06$ eV and $M_1 = 10^{11}$ GeV is given in the last column.

The four different temperature regimes that we consider are distinguished by the additional interactions that enter into equilibrium as the temperature of the thermal bath decreases. Of course, the most important of these reactions will be those mediated by the charged lepton Yukawa couplings.

1) *Bottom- and tau-Yukawa interactions in equilibrium* (10^{12} GeV $\lesssim T \lesssim 10^{13}$ GeV).

Equilibrium for h_b - and h_τ -interactions implies that the asymmetries in the $SU(2)$ singlet b and e_τ degrees of freedom are populated. The corresponding chemical potentials obey the equilibrium constraints $\mu_b = \mu_{Q_3} - \mu_H$ and $\mu_\tau = \mu_{\ell_\tau} - \mu_H$. Possibly h_b and h_τ Yukawa interactions enter into equilibrium at a similar temperature as the electroweak sphalerons [24]. However, since the rate of the non-perturbative processes is not well known, we first consider the possibility of a regime with only gauge, QCD sphaleron and the Yukawa interactions of the whole third family in equilibrium. Since electroweak sphalerons are not active, at this stage $B = 0$.

The lepton ($\ell_a, \ell_b, \ell_\tau$) and antilepton ($\bar{\ell}'_a, \bar{\ell}'_b, \bar{\ell}'_\tau$) flavor bases are defined such that $\langle \ell_1 | \ell_b \rangle = \langle \bar{\ell}'_1 | \bar{\ell}'_b \rangle = 0$, and hence $|\langle \ell_1 | \ell_a \rangle|^2 \equiv K_a = 1 - K_\tau$ and $|\langle \bar{\ell}'_1 | \bar{\ell}'_a \rangle|^2 \equiv \bar{K}_a = 1 - \bar{K}_\tau$. The ‘prime’ labeling the a and b antilepton doublets is a reminder of the fact that in general $\bar{\ell}'_{a,b}$ are not the CP conjugate states of $\ell_{a,b}$. Even though the charged lepton Yukawa interactions for both a and b states are negligible, the fact that $L_b = \rho_{bb'} = 0$ ensures that the off-diagonal entries $\rho_{ab'}$ and $\rho_{ba'}$ vanish. Given that $Y_{\Delta_b}/Y^{eq} = 2y_{\ell_b} = 0$, the set of eqs. (4.6) is reduced to just two relevant equations, and the equilibrium conditions

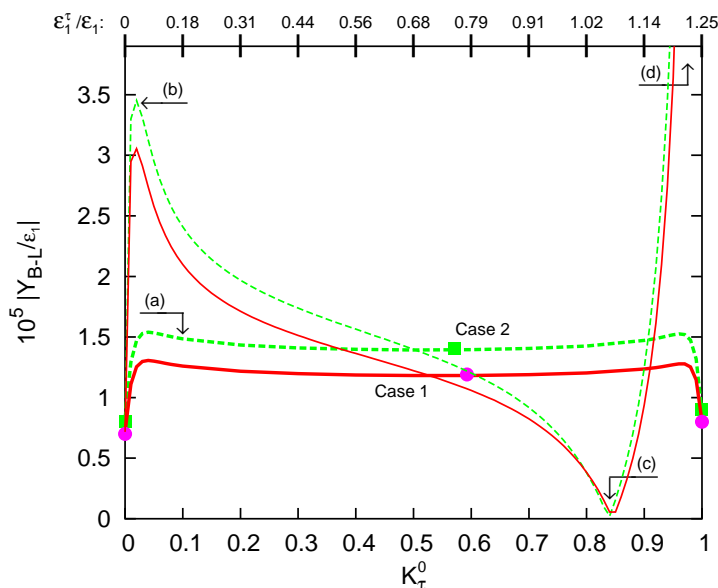


Figure 1: The value of the final $|Y_{B-L}|$ (in units of $10^{-5}|\epsilon_1|$) as a function of K_τ^0 in the regimes 1 (solid lines) and 2 (dashed lines), computed with $M_1 = 10^{11}$ GeV and $\tilde{m}_1 = 0.06$ eV. The filled circles (regime 1) and squares (regime 2) give the aligned cases ($K_\tau^0 = 0, 1$) discussed in ref. [13] and the non-aligned cases given in the first two rows in table 1. The thick lines correspond to non-aligned models for which $\Delta K_\tau = 0$, implying $\epsilon_1^\tau/\epsilon_1 = K_\tau^0$. The thin lines give an example of the results for $K_\tau \neq \bar{K}_\tau$ assuming $\Delta K_\tau/2\epsilon_1 = \sqrt{K_\tau^0}/4$. The corresponding values of $\epsilon_1^\tau/\epsilon_1$ are marked on the upper x -axis. The arrows with labels (a),(b),(c) and (d) correspond to the four panels in figure (2). Note that Y_{B-L} changes sign in (c).

(restricted to the a, τ subspace) lead to the following values for C_i^H and C_{ij}^ℓ :

$$C^H = \frac{1}{16}(3, 4) \quad \text{and} \quad C^\ell = \frac{1}{32} \begin{pmatrix} 16 & 0 \\ 1 & 12 \end{pmatrix}. \quad (4.8)$$

The effective one-flavor approximation to non-alignment for this regime (the first line in table 1) corresponds to $K_\tau^0 = 16/27$, as follows from the condition $\sum_i K_i C_{ij}^\ell = \tilde{c}_\ell$ ($i, j = a, \tau$). This value ensures $\tilde{y} \propto Y_{B-L}$, giving in this case $\tilde{c}_\ell = 2/9$ (while $\tilde{c}_H = \sum_{j=a,\tau} C_j^H/2 = 7/32$). The enhancement of $|Y_{B-L}|$ in our representative case is of about 60% with respect to the aligned cases [13] in the same temperature regime.

In figure 1 we display by a thick solid line the $B - L$ asymmetry as a function of K_τ^0 . This curve corresponds to $K_\tau = \bar{K}_\tau$ and is obtained by numerical integration of the system of equations (4.6). Three special cases are denoted with filled circles: The flavor-dependent case with $K_\tau^0 = 16/27$ (described in the table) and the flavor-aligned cases $K_\tau^0 = 0, 1$ (discussed in [13]). Two features that are expected from the qualitative discussion in section 2.2 are apparent in this figure: First, the effects of flavor misalignment are quite insensitive to the particular value of K_τ^0 (as long as it is not too close to the alignment conditions). Second, the one-flavor approximation (the filled circle at $K_\tau^0 = 16/27$) provides a good estimate of Y_{B-L} for generic values of K_τ^0 .

In the more general cases where $K_\tau \neq \bar{K}_\tau$, several different parameters concur to determine the final value of $B-L$. One way to explore the possible results in this case would be to randomly sample the values of the Yukawa couplings and CP violating phases entering the expressions for $K_{\tau,a}^0$, $\Delta K_{\tau,a}$ and ϵ_1 and present the results in the form of scatter plots. However, to gain a qualitative understanding of the various possibilities, we proceed in a simpler way. It is based on the fact that, if we take the tree-level $N_1 \rightarrow \ell_\tau H$ decay amplitude to zero ($\lambda_{1\tau} \rightarrow 0$) while keeping the total decay rates fixed [$(\lambda\lambda^\dagger)_{11} = \text{const.}$], the τ -flavor projector K_τ^0 vanishes as the square of this amplitude ($\propto |\lambda_{1\tau}|^2$) while ΔK_τ vanishes as the amplitude ($\propto \lambda_{1\tau}$). This suggests to adopt, as a convenient ansatz, the relation $\Delta K_\tau \propto \sqrt{K_\tau^0}$ also for finite values of K_τ^0 , allowing us to explore the interplay between the contributions of ΔK_τ and of $\epsilon_1 K_\tau^0$ to ϵ_1^τ — see eq. (2.7) — as well as to $\epsilon_1^a = \epsilon_1 - \epsilon_1^\tau$, by means of a simple two-dimensional plot. To this aim, we take $\Delta K_\tau/2\epsilon_1 = \kappa_\tau \sqrt{K_\tau^0}$ and we fix κ_τ to the representative value of $1/4$. Thus, for the practical purpose of carrying out a qualitative survey of the possible different situations, we adopt the following ansatz:

$$\text{ansatz :} \quad \frac{\epsilon_1^\tau}{\epsilon_1} = K_\tau^0 + \frac{1}{4} \sqrt{K_\tau^0}. \quad (4.9)$$

With respect to the hierarchy between the two different CP violating effects of eqs. (2.1) and (2.2), this simple relation has the nice property of covering all the interesting possibilities:

1. For $K_\tau^0 \lesssim 1/16$, the two terms $K_\tau^0 \epsilon_1$ and ΔK_τ are comparable in size.
2. For $1/16 \lesssim K_\tau^0 \lesssim 3/4$, $K_\tau^0 \epsilon_1$ and $K_a^0 \epsilon_1$ dominate the respective asymmetries.
3. For $K_\tau^0 > 3/4$, we enter a regime in which $\Delta K_\tau/2 \gg K_a^0 \epsilon_1$ and, since $\Delta K_a = -\Delta K_\tau$, the asymmetry ϵ_1^a , that is largely dominated by the ΔK_a term, has now the opposite sign with respect to ϵ_1 and ϵ_1^τ .

In figure 1 we show with the thin continuous line the results for this more general case. On the upper x -axis we have marked for reference the value of $\epsilon_1^\tau/\epsilon_1$ corresponding to each different value of K_τ^0 . The most peculiar features are the two narrow regions marked with (b) and (d) where $|Y_{B-L}|$ is strongly enhanced. The enhancement takes place at values of K_τ^0 not far from alignment. In particular, the steep rise of $|Y_{B-L}/\epsilon_1|$ close to $K_\tau^0 \approx 1$ can reach values up to one order of magnitude larger than the vertical scale of the figure. (Note, however, that the ansatz eq. (4.9) does not yield the required behavior $\Delta K_a = -\Delta K_\tau = 0$ for $K_a^0 = 1 - K_\tau^0 = 0$, and therefore the thin lines in the plot should not be extrapolated to $K_\tau^0 \simeq 1$.) It is worth noticing that that the two peaks correspond to values of Y_{B-L}/ϵ_1 of opposite sign, since the asymmetry changes sign in (c). Qualitatively similar effects occur in the lower temperature regimes discussed below, and therefore we postpone the analysis of these results to section 5. As concerns the quality of our model independent approximation, we see from figure 1 that, in this more general case, it can provide a reasonable estimate of Y_{B-L} only for $K_\tau^0 \sim K_a^0 \sim 1/2$, that is when N_1 decays with similar rates into ℓ_τ and ℓ_a .

Given that the final value of Y_{B-L} depends linearly on ϵ_1^τ (see eq. 2.8), we see from eq. (2.7) that, for a fixed value of K_τ^0 , Y_{B-L} is linear in ΔK_τ . Therefore, from the two thick and thin lines in figure 1 that respectively give Y_{B-L}/ϵ_1 for $\Delta K_\tau = 0$ and for $\Delta K_\tau/(2\epsilon_1) = \kappa_\tau \sqrt{K_\tau^0}$, one can easily infer, for each K_τ^0 , the value of Y_{B-L}/ϵ_1 corresponding to any other value of ΔK_τ . In particular, by reflecting the thin line with respect to the thick one, one can figure out the results one would obtain for $\kappa_\tau < 0$.

2) *Electroweak sphalerons in equilibrium* (10^{11} GeV $\lesssim T \lesssim 10^{12}$ GeV).

The electroweak sphaleron processes take place at a rate per unit volume $\Gamma/V \propto T^4 \alpha_W^5 \log(1/\alpha_W)$ [27–29], and are expected to be in equilibrium from temperatures of about $\sim 10^{12}$ GeV, down to the electroweak scale or below [24]. Electroweak sphalerons equilibration implies

$$\sum_i (3\mu_{Q_i} + \mu_{\ell_i}) = 0. \quad (4.10)$$

As concerns lepton number, each electroweak sphaleron transition creates all the doublets of the three generations, implying that individual lepton flavor numbers are no longer conserved, regardless of the particular direction in flavor space along which the doublet ℓ_1 and $\bar{\ell}'_1$ lie. While the electroweak sphalerons induce $L_b \neq 0$, the condition $\Delta_b = 0$ is not violated, and hence eqs. (4.6) still consist of just two equations for Y_{Δ_a} and for Y_{Δ_τ} . As concerns baryon number, electroweak sphalerons are the only source of B violation, implying that baryon number is equally distributed among the three quark generations, that is $B/3$ in each generation. This modifies the detailed equilibrium conditions for the quark chemical potentials. Besides these differences, the analysis follows closely the one carried out in the previous regime. The coefficients C_i^H and C_{ij}^ℓ are given by

$$C^H = \frac{1}{230}(41, 56) \quad \text{and} \quad C^\ell = \frac{1}{460} \begin{pmatrix} 196 & -24 \\ -9 & 156 \end{pmatrix}. \quad (4.11)$$

In table 1 we give the values of \tilde{c}_ℓ and \tilde{c}_H that correspond to the approximation in eq. (4.7). For the models described by this approximation, flavor misalignment can induce an $\mathcal{O}(80\%)$ enhancement of the final $B-L$ asymmetry compared to the aligned cases discussed in [13]. The source of this enhancement is mainly the suppression by a factor $\sim n_f = 2$ of the washout processes that is apparent in the reduced value of \tilde{c}_ℓ . In figure 1 we give the final value of $|Y_{B-L}|$ as a function of K_τ^0 for $\Delta K_\tau = 0$ (thick dashed line) and for $\Delta K_\tau \neq 0$ (thin dashed line). The qualitative dependence of Y_{B-L} on K_τ^0 is quite similar to regime 1: If the only CP violating effects are of the type in eq. (2.1) ($CP(\bar{\ell}'_1) = \ell_1$), the results are approximately independent of the particular value of K_τ^0 , while relaxing this condition results again in a strong dependence on K_τ^0 and possibly strong enhancements. This general case is again well approximated by the values of \tilde{c}_ℓ and \tilde{c}_H given in the table when $K_\tau^0 \sim K_a^0 \sim 1/2$.

3) *Second generation Yukawa interactions in equilibrium* (10^8 GeV $\lesssim T \lesssim 10^{11}$ GeV).

In this regime, h_c , h_s and, most importantly, h_μ Yukawa interactions enter into equilibrium. Given that the electron remains the only lepton with a negligible Yukawa coupling, the Yukawa interactions completely define the flavor basis for the leptons as well

as for the antileptons (that are now the CP conjugate states of the leptons). Correspondingly, the lepton asymmetries are also completely defined in the flavor basis. In this regime, the coefficients C_{ij}^ℓ and C_i^H projecting the asymmetries ($y_{\ell_e}, y_{\ell_\mu}, y_{\ell_\tau}$) and y_H onto $(Y_{\Delta_e}, Y_{\Delta_\mu}, Y_{\Delta_\tau},)$ are:

$$C^H = \frac{1}{358}(37, 52, 52) \quad \text{and} \quad C^\ell = \frac{1}{2148} \begin{pmatrix} 906 & -120 & -120 \\ -75 & 688 & -28 \\ -75 & -28 & 688 \end{pmatrix}. \quad (4.12)$$

For the approximation of eq. (4.7), the coefficient \tilde{c}_ℓ in table 1 is reduced by at least a factor of three with respect to the values of c_ℓ in the aligned cases analyzed in [13], and the final value of Y_{B-L} gets correspondingly enhanced. This is precisely the $n_f \sim 3$ enhancement expected from the qualitative discussion in section 2.2. The estimate of Y_{B-L} based on the one-flavor approximation is, again, rather precise for the class of models for which $\Delta K_i = 0$, independently of the particular values of $K_{e,\mu,\tau}^0$ (as long as they are not too close to 0 or 1, where approximate alignment could induce effects that suppress Y_{B-L}). In the more general case with $\Delta K_{e,\mu,\tau} \neq 0$, the approximation is again reliable in the region of approximately equal flavor composition for ℓ_1 and $\bar{\ell}'_1$, that is around $K_e^0 \sim K_\mu^0 \sim K_\tau^0 \sim 1/3$. As regards regions in K space away from equal flavor composition, we expect that the enhancements observed in the $n_f = 2$ cases, that are especially large when one K_i^0 is small (see figure 1), could be even larger in this case since now two flavor projectors can be simultaneously small.

4) *All SM Yukawa interactions and electroweak sphalerons in equilibrium ($T \ll 10^8$ GeV).* Equilibration of the Yukawa processes for all quarks and leptons, including the electron, occurs only at temperatures $T < 10^6$ GeV, which are too low to be relevant for leptogenesis (at least in the standard scenarios). We nevertheless analyze this regime, mainly for the purpose of comparison with analysis that assume that all Yukawa interactions are in equilibrium, but do not include the flavor effects [12, 13].

The coefficients C^H and C^ℓ are given by

$$C^H = \frac{8}{79}(1, 1, 1) \quad \text{and} \quad C^\ell = \frac{1}{711} \begin{pmatrix} 221 & -16 & -16 \\ -16 & 221 & -16 \\ -16 & -16 & 221 \end{pmatrix}. \quad (4.13)$$

Given the symmetric situation of having all Yukawa interactions in equilibrium, the point of equal flavor composition for the $\ell_1^{(j)}$ doublets ($K_e = K_\mu = K_\tau = 1/3$) defines a fully flavor-symmetric situation for which $Y_{\Delta_\tau} = Y_{\Delta_\mu} = Y_{\Delta_e} = Y_{B-L}/3$. It is then straightforward to see that this point corresponds to the condition $\tilde{y}_\ell = \sum_i K_i^0 y_{\ell_i} = -\tilde{c}_\ell Y_{B-L}/Y^{eq}$ that defines our one-flavor approximation. The symmetric situation implies that an exact proportionality is also obtained for $y_H = -\tilde{c}_H Y_{B-L}/Y^{eq}$ (independent of the particular values of $K_{e,\mu,\tau}^0$). In agreement with the qualitative analysis in section 2.2, flavor effects suppress \tilde{c}_ℓ by the large factor 3.5 ($\sim n_f$) compared to the corresponding aligned case [13]. In the cases restricted by $\Delta K_i = 0$ for which the results of the one-flavor approximation hold, the final value of Y_{B-L} is enhanced by a factor ~ 2.5 with respect to the aligned case [13]. Again, much larger enhancements are possible if the condition $K_i = \bar{K}_i$ is relaxed.

5. Discussion

In this section we explain some generic features of our results. We refer here to results that, while depending on the specific flavor structures, are qualitatively similar in all the temperature regimes in which flavor effects are important. For the purposes of this discussion, it is important the fact that in all the regimes considered the final Y_{Δ_i} asymmetries are inversely proportional to the rates of the corresponding washout processes. This can be demonstrated along lines similar to those given in appendix 2 of ref. [14] for Y_{B-L} . Note that we have already used this result extensively for the qualitative discussion in section 2.2. In all the temperature regimes where flavor effects are important ($M_1 \ll 10^{13}$ GeV), the washout rate having the strongest impact on the final value of the asymmetries is γ_{N_s} , that is the on-shell piece of the $\Delta L = 2$ scattering (the term proportional to $K_i^0 \gamma_D$ in the first line of eq. (3.2)) that has a Boltzmann suppression factor $\exp(-z)$, similar to the $\Delta L = 1$ rates. Hence, the proof given in ref. [14] for the case of $\Delta L = 1$ washout dominance and small departure from equilibrium, holds also for the cases considered here, and applies to the single flavor density asymmetries as well.

As is written explicitly in eq. (2.8), the final values of the Y_{Δ_i} are determined by the asymmetry parameters ϵ_1^i and by the washout factors η_i . The washout factors are related to the various lepton number violating processes of eq. (4.6) and hence depend on the factors K_i^0 that control the overall strength of the washouts, as well as on the matrix coefficients C^ℓ and C^H defined in (4.5). However, the matrices C^ℓ in eqs. (4.8), (4.11), (4.12) and (4.13) are approximately diagonal and the diagonal terms in each matrix are not very different from each other (actually, for each temperature regime, C_{ii}^ℓ equals the value of c_ℓ in the case that $K_i^0 = 1$, see [13]). If we make the crude approximations of (i) neglecting the off-diagonal elements, and (ii) taking the diagonal elements equal (that is $C^\ell \propto 1$), and we also note that the same Higgs asymmetry y_H enters the equation for the different flavors, we are led to conclude that the relative values of the η_i are determined mainly by the values of the K_i^0 . More precisely, given that the amount of flavor-asymmetries surviving the washout are, to a good approximation, inversely proportional to the washout rates, the washout factors obey the approximate proportionality $\eta_i \propto 1/K_i$. This results constitutes the basis of the approximate expressions for n_{B-L}/s in eq. (2.9) that allowed us to estimate qualitatively all the most important effects of flavor. We rely again on this approximation in the following discussion of the effects of different flavor structures and, in particular, in our attempt to understand the different behaviors that are apparent from the points labeled (a), (b), (c) and (d) in figure 1.

(a) This case is defined by the condition $K_i = \bar{K}_i$ or, more generally, by the condition $\Delta K_i \ll \epsilon_1 K_i^0$ for all relevant i . As we have seen, in this case the final Y_{B-L} shows a sizable enhancement with respect to the aligned cases in the same temperature regime, and its value is approximately independent of K_i^0 . This behavior corresponds to the $\sim n_f$ enhancement discussed in section 2.2: since $K_i = \bar{K}_i$ implies $\epsilon_1^i = K_i^0 \epsilon$, it follows that, to first approximation, each $Y_{\Delta_i} \propto \epsilon_1^i \eta_i$ ($i = a, \tau$ in regimes (1) and (2), $i = e, \mu, \tau$ in regimes (3) and (4)) is independent of K_i^0 and, moreover, that the final values of the Y_{Δ_i} -asymmetries are in general quite similar. The enhancement of Y_{B-L} by a factor $\sim n_f$

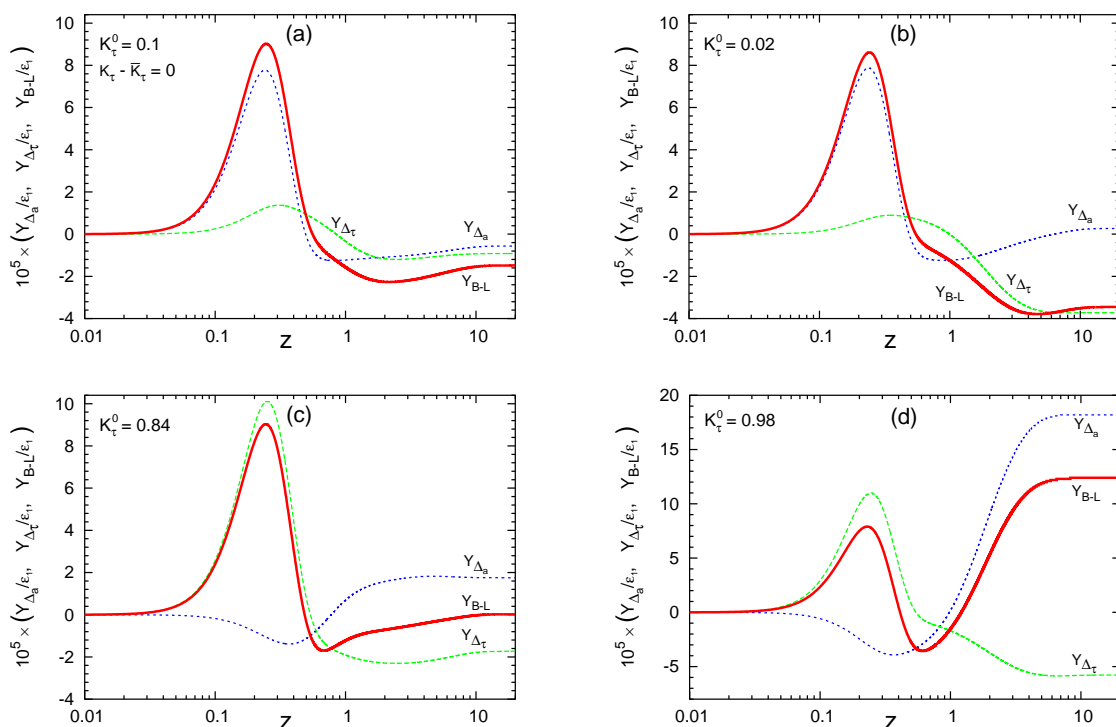


Figure 2: $Y_{\Delta_\tau}/\epsilon_1$ (short-dashed line), Y_{Δ_a}/ϵ_1 (dotted line) and Y_{B-L}/ϵ_1 (solid line) as a function of $z = M_1/T$ computed with $M_1 = 10^{11}$ GeV and $\tilde{m}_1 = 0.06$ eV, in the temperature regime 2. The panels refer to the labels in figure 1. The examples include both a constrained case, $\Delta K_\tau = 0$ (a), and general cases with $\Delta K_\tau/2\epsilon = \sqrt{K_\tau^0}/4$ (b, c, d). The K_τ^0 value is (a) 0.1, (b) 0.02, (c) 0.84 and (d) 0.98. Note that the vertical scale in (d) is doubled with respect to the other three cases.

with respect to the aligned cases [13] is indeed confirmed by the numerical results given in table 1 ($n_f = 2$ in regimes (1) and (2), $n_f = 3$ in regimes (3) and (4)). This case is illustrated in more detail in figure 2a. The figure shows the evolution of Y_{Δ_a} and Y_{Δ_τ} with z in the regime 2, for $K_\tau^0 = 0.1$. It is apparent that in spite of the large difference in the values of K_a^0 and K_τ^0 , that result in quite different evolutions for the two asymmetries, their final values are very similar and approximately equal to $Y_{B-L}/2$.

Note that this result relies in a crucial way on the assumption that the source terms ϵ_1^i and the washout rates are both proportional to K_i^0 . If this condition is not realized, as in the general cases when $K \neq \bar{K}$, more subtle effects become important in determining the final values of the asymmetries Y_{Δ_i} and, as is apparent from figure 1, the final value of Y_{B-L} acquires a rather strong dependence on the values of K_i^0 . Let us now discuss a few examples of this more general case.

(b) In this case, $\Delta K_\tau/2$ and $\epsilon_1 K_\tau^0$ are of the same order and of the same sign. The details of this case are illustrated in figure 2b where we present the behavior of Y_{Δ_τ} and Y_{Δ_a} for $K_\tau^0 = 0.02$, that corresponds to $\Delta K_\tau/2\epsilon = \sqrt{K_\tau^0}/4 \simeq 0.035$. The picture explains rather well the origin of the first peak, labeled with (b) in figure 1. We see that a deviation at the level of just a few percent from exact alignment ($K_\tau^0 = 0$) is sufficient to start populating the

asymmetry in the τ flavor even if the rate, which is suppressed by $K_\tau^0 + \sqrt{K_\tau^0}/4 \sim 1/20$, is rather small. For the first part of the leptogenesis era, Y_{Δ_τ} gives only a minor contribution to the total $B - L$ asymmetry. However, while the washout in the direction $\ell_a (\perp \ell_\tau)$, that is controlled by $K_a^0 \approx 1$, proceeds with full strength, there is practically no washout for the τ component and, eventually, while Y_{Δ_a} ends up being almost completely washed out, it is Y_{Δ_τ} that determines the final value of $B - L$ at the end of the leptogenesis era.

(c) This intriguing case corresponds to $\epsilon_1^\tau \eta_\tau \approx -\epsilon_1^a \eta_a$: The final values of the two leptonic asymmetries are approximately equal in magnitude but of opposite signs. This implies that Y_{B-L} can vanish even when lepton flavor asymmetries are sizable. Eq. (2.8) helps us to understand this case in a simple way. Adopting the approximation $\eta_i \sim \eta/K_i^0$ discussed above we have:

$$\epsilon_1^a \eta_a + \epsilon_1^\tau \eta_\tau \approx \eta \left[2\epsilon_1 - \left(\frac{1}{1 - K_\tau^0} - \frac{1}{K_\tau^0} \right) \frac{\Delta K_\tau}{2} \right]. \quad (5.1)$$

With our particular ansatz, $\Delta K_\tau/2\epsilon_1 = \sqrt{K_\tau^0}/4$, the r.h.s. of eq. (5.1) vanishes for $K_\tau^0 \simeq 0.9$, in qualitative agreement with the point labeled with (c) in figure 1. The evolution with z of the flavor asymmetries is illustrated in figure 2c. During most of the leptogenesis era, Y_{B-L} is rather large. However, in the end its value is driven to zero, even if the final values of the two lepton asymmetries remain quite sizable (about twice larger than case (a)). For larger values of K_τ^0 the total asymmetry Y_{B-L} crosses zero and changes sign, and we enter a different regime to be discussed next.

(d) In this example we have $\Delta K_a/2\epsilon_1 \simeq -0.25$ and $K_a^0 = 0.02$, so that $\Delta K_a/2\epsilon_1$ is much larger in absolute value than K_a^0 , and is negative. This situation means that ϵ_1^a has a sign opposite to both ϵ_1^τ and ϵ_1 . This case is illustrated in figure 2d, in which the vertical scale is doubled with respect to the previous three cases. Initially, a large density-asymmetry starts being built in the τ flavor, due to the large value $\epsilon_1^\tau/\epsilon_1 \sim 1.23$. However, its growth is kept under control by the washout effects that, with $K_\tau^0 \approx 1$, are unsuppressed. In contrast, for Y_{Δ_a} the washout effects are strongly suppressed by $K_a^0 \sim 0.02$ and eventually this asymmetry largely prevails, driving Y_{B-L} to values several times larger than in all the previous cases, and having opposite sign. Notice that in this case the difference in sign between Y_{Δ_a} and Y_{Δ_τ} gives large cancellations between the respective contributions to the asymmetry y_H , and this implies that the Higgs washout effects are accordingly suppressed.

6. Conclusions

We have shown that the effects of flavor can have dramatic consequences in leptogenesis scenarios. This occurs due to the way in which charged lepton Yukawa interactions in thermal equilibrium affect the flavor composition of the leptonic density-asymmetries, that determine the washout processes. For these effects to be significant, at least one leptonic Yukawa interaction (h_τ) must be in equilibrium. This happens if M_1 , the lightest heavy neutrino mass that determines the temperature at which leptogenesis takes place, is light enough: $M_1 < 10^{13}$ GeV. Moreover, for the flavor effects to have an impact, one needs to be in the strong washout regime: $\tilde{m}_1 > 2 \times 10^{-3}$ eV.

The main consequence of the flavor effects is that, in generic flavor non-aligned cases with strong washouts, the final asymmetry is typically enhanced by a factor n_f . We have $n_f = 2$ when just h_τ is in equilibrium, i.e. for $10^9 \text{ GeV} < M_1 < 10^{13} \text{ GeV}$, and $n_f = 3$ when also h_μ is in equilibrium, i.e. for $M_1 < 10^9 \text{ GeV}$.

In addition to the total asymmetry ϵ_1 associated to N_1 decays, the individual asymmetries ϵ_1^j can play a crucial role. These flavor asymmetries introduce a qualitatively new ingredient, ΔK_j , which is the contribution associated to the fact that the leptons and antileptons produced in N_1 decays are, in general, not CP conjugates of each other. When these new contributions are non-negligible, much larger enhancements become possible, especially when at least one lepton flavor ℓ_j is weakly coupled to the decaying N_1 (typically this means $K_j^0 \sim \eta \ll 1$). Since the signs of ΔK_j and of ϵ_1 need not be the same, the final asymmetry can have a sign unrelated to that of ϵ_1 . Actually, successful leptogenesis is possible even with $\epsilon_1 = 0$. Scenarios in which $\epsilon_1 = 0$ while $\epsilon_1^j \neq 0$ entail the possibility that the phases in the light neutrino mixing matrix U are the only source of CP violation.

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